1990-06

Robustness of an adaptive pole placement algorithm in the presence of bounded disturbances and slow time variation of parameters

Das, M.; Cristi, R.

IEEE

M. Das, R. Cristi, "Robustness of an adaptive pole placement algorithm in the presence of bounded disturbances and slow time variation of parameters," IEEE Transactions on Automatic Control, v. 35, no.6, (June 1990), pp. 752-756.

http://hdl.handle.net/10945/56654

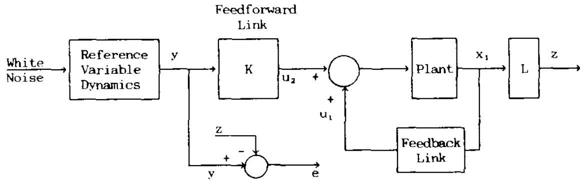

Fig. 4. Tracking system.

Fig. 4. The dynamical model, after using a control of the form (5), is

$$\dot{x}_1 = A_{11}x_1 + BKCx_2 \tag{25}$$

where it is assumed that the model to be tracked is not coupled to x_1 initially, so that $\hat{A}_{12} = 0$. The tracking error is

$$e = y - z = Cx_2 - Lx_1 \tag{26}$$

so that Lx_1 is to be made to track y.

The performance measure is assumed to be quadratic:

$$J = \mathrm{tr}\left\{\int_{t_0}^{t_f}[e^TQe + u^TRu]\,dt + e^T(t_f)Se(t_f)\right\}. \tag{27}$$

This can be written as

$$\begin{aligned} J = \mathrm{tr}\Bigg\{\int_{t_0}^{t_f} & [C^TQCP_{22} + L^TQLP_{11} - C^TQLP_{12} \\ & - L^TQCP_{21} + C^TK^TRKCP_{22}]\,dt \\ & + C^TSCP_{22}(t_f) + L^TSLP_{11}(t_f) \\ & - C^TSLP_{12}(t_f) - L^TSCP_{21}(t_f)\Bigg\} \end{aligned} \tag{28}$$

where

$$\begin{aligned} \dot{P}_{11} &= A_{11}P_{11} + BKCP_{21} + P_{12}C^TK^TB^T + P_{11}A_{11}^T \\ \dot{P}_{12} &= A_{11}P_{12} + BKCP_{22} + P_{12}A_{22}^T \end{aligned} \tag{29}$$

and P_{22} is as in (8).

Solving the problem for the optimal K is straightforward, as in the previous section. The gain K satisfies (11) with

$$\dot{\Lambda}_{11} = -L^TQL - A_{11}^T\Lambda_{11} - \Lambda_{11}A_{11} \tag{30}$$

$$\dot{\Lambda}_{12} = -L^TQC - \Lambda_{11}BKC - A_{11}^T\Lambda_{12} - \Lambda_{12}A_{22}. \tag{31}$$

The boundary conditions for (30) and (31) are

$$\begin{aligned} \Lambda_{11}(t_f) &= L^TSL \\ \Lambda_{12}(t_f) &= -L^TSC. \end{aligned} \tag{32}$$

It is clear that again we have P_{22} and Λ_{11} precomputable, so that K is just a linear function of P_{12} and Λ_{12}.

Equations (12) and (14) apply with the following changes:

$$\theta_1 = 0 \quad \theta_2 = -L^TQC.$$

Thus, with minor changes in the driving terms and in the boundary conditions, the same equations (18) apply to both the tracking and regulation problems.

Remarks and Conclusion

We have considered the idea of not using full state information in controlling a large system. This could be motivated by either the fact that the full state is not available or that it is desirable to reduce the number of real-time multiplications needed in implementing the controller. The class of problems we have considered is often referred to as output feedback problems, and within this context, we have focused on two problems with an uncontrollable portion, the tracking and disturbance rejection problems. Because they are an important class of problems, and because their solution is reasonably straightforward, we have felt that they deserved attention. In particular, the solution for the optimal gains only involves a linear two-point boundary-value problem. However, it should be noted that the off-line design calculation requirements may be considerably more severe than is true for the full state feedback case. The invocation of the Kronecker algebra clearly indicates this fact. Nevertheless, the situation is far better than having a nonlinear matrix two-point boundary-value problem to solve.

The reader may wonder about some extensions to the results presented here. The discrete case is easily handled with no real surprises in the mathematics required. The case of noisy measurements can be handled by designing a reduced-order filter to estimate y, as in [4]. The state x_2 is then augmented to include the filter states, and one can proceed with the methodology presented here. However, it should be noted that the reduced-order filter gains affect the control gains, so there is no separation theory here in the reduced-order setting.

References

[1] H. Kwakernak and R. Sivan, *Linear Optimal Control Systems*. New York: Wiley-Interscience, 1972.

[2] S. Axsater, "Suboptimal time variable feedback control of linear dynamic systems with random inputs," *Int. J. Contr.*, vol. 4, no. 6, pp. 549–566, 1966.

[3] J. W. Brewer, "Kronecker products and matrix calculus in systems theory," *IEEE Trans. Circuits Syst.*, vol. CAS-25, Sept. 1978.

[4] C. S. Sims, "Reduced order modeling and filtering," in *Advances in Control and Dynamic Systems*, vol. 18, C. T. Leondes, Ed. New York: Academic, 1982, pp. 55–103.

Robustness of an Adaptive Pole Placement Algorithm in the Presence of Bounded Disturbances and Slow Time Variation of Parameters

M. DAS AND R. CRISTI

Abstract—**The purpose of this note is to study the robustness properties of a specific adaptive pole placement scheme in the presence of bounded disturbances and slow time variation of parameters. We show that the least squares (with block processing) algorithm used in this scheme has the remarkable property of retaining the global stability of the overall system in the presence of bounded disturbances of small magnitude and slow time variation of system parameters (within a compact set).**

I. Introduction

The past few years have seen a significant growth in the literature on robust adaptive control theory. As a result of the concerted efforts of many researchers in this field [1]–[14], robustness problems have now been clearly defined, and many practically applicable solutions have emerged.

All the research results until this date can be broadly categorized into two groups. Within the first group, one may include the techniques proposed for robustification of the existing adaptive control algorithms [1], and also a class of new robust adaptive control schemes [2]–[5]. An excellent unified treatment of most of these methods can be found in [2]. Within the second group, one may include all the other results [6]–[14]

Manuscript received October 14, 1988; revised July 17, 1989.

M. Das is with the Department of Electrical and Systems Engineering, Oakland University, Rochester, MI 48309.

R. Cristi is with the Department of Electrical and Computer Engineering, Naval Postgraduate School, Monterey, CA 93943.

IEEE Log Number 9035073.

0018-9286/90/0600-0752$01.00 © 1990 IEEE

which emphasize the robustness properties of the existing adaptive control schemes. An excellent summary of most of these methods can be found in [6].

The result presented in this note falls into the second category as defined above. Of particular interest to us is the study of the robustness properties of the least squares (with block processing) algorithm that was introduced by Elliott *et al.* [14] in the context of adaptive pole placement, and modified later by Elliott and Goodwin [5] for adaptive implementation of the internal model principle. In this note, we study the robustness of this algorithm with respect to bounded random disturbances and slow time variation of system parameters.

The organization of this note is as follows. Section II summarizes the system modeling aspects and formulation of the problem under study. In Section III, we present a global stability analysis. Finally, in Section IV, we present some concluding remarks.

II. System Modeling Assumption and Problem Formulation

Consider the problem of indirect adaptive control of a linear, time-invariant, discrete-time system characterized by the following difference equations:

$$A(q^{-1})z(t) = u(t) + w_i(t) \tag{1a}$$

$$y(t) = B(q^{-1})z(t) + w_o(t). \tag{1b}$$

Here, $w_i(t)$ and $w_o(t)$ may represent either the signals which arise due to time variation of system parameters, i.e., coefficients of $A(q^{-1})$ and $B(q^{-1})$, or these may represent bounded random disturbances at the input and output ends, respectively.

In the analysis presented below, it will be found convenient to rewrite (1a)–(1b) in the following compact form:

$$A(q^{-1})y(t) = B(q^{-1})u(t) + w(t) \tag{2a}$$

where

$$w(t) = B(q^{-1})w_i(t) + A(q^{-1})w_o(t) \tag{2b}$$

or, equivalently,

$$y(t) = \phi(t-1)^T\theta + w(t) \tag{2c}$$

where T denotes transpose and

$$\theta = [-a_1, -a_2, \cdots, -a_n, b_1, b_2, \cdots, b_n]^T \tag{2d}$$

$$\phi(t-1) = [y(t-1), y(t-2), \cdots, y(t-n), u(t-1), u(t-2), \cdots, u(t-n)]^T. \tag{2e}$$

Consider next the application of the adaptive control law:

$$L_k(q^{-1})u(t) = P_k(q^{-1})(r(t) - y(t)), \qquad t \in [kN, (k+1)N-1]. \tag{3}$$

Here, the coefficients of the polynomials $A(q^{-1})$ and $B(q^{-1})$ are estimated by using the least squares with covariance resetting algorithm [15], [5]. $A_k(q^{-1})$ and $B_k(q^{-1})$ denote the estimated coefficients at time $t = kN$ where N denotes the block size. The polynomials $L_k(q^{-1})$ and $P_k(q^{-1})$ are of degree n and their coefficients are computed from

$$L_k(q^{-1})A_k(q^{-1}) + P_k(q^{-1})B_k(q^{-1}) = A^*(q^{-1}) \tag{4}$$

where $A^*(q^{-1})$ is an arbitrary Hurwitz polynomial whose zeros represent the desired closed-loop pole locations.

III. Stability Analysis

Notice that the closed-loop system is characterized by the equations

$$(L_kA + P_kB)z(t) = P_kr(t) + L_kw_i(t) - P_kw_o(t) \tag{5a}$$

$$Rr(t) = 0, \qquad t \in [kN, (k+1)N-1], \tag{5b}$$

where, for notational convenience, we dropped the argument q^{-1} from all the polynomials.

The following simplifying assumptions are made in carrying out the stability analysis of the closed-loop system.

1) The polynomials A and B are assumed to be relatively prime.

2) The reference $r(t)$ contains $m \geq 3n$ complex sinusoidal components.

3) The block size N is chosen to be greater than or equal to $4n$.

4) If random disturbance is present, we assume that for all t, $|w_i(t)| < \delta$, $i = 1, 2$.

5) If the coefficients of A and B vary with time, these variations are assumed to occur infrequently and remain confined to a small set of operating points S defined by

$$S = \{\theta_i | \theta_i = [-a_{i1}, \cdots, -a_{in}, b_{i1}, \cdots, b_{in}]^T \text{ with coprime } A_i(q^{-1}) \text{ and } B_i(q^{-1})\}.$$

Remark 1: For the case of a time-varying system, define the diameter δ of S by

$$\delta = \max_{\theta_i, \theta_j \in S} \|\theta_i - \theta_j\| \tag{6a}$$

and designate a center point $\theta_c \in S$ such that

$$\|\theta_i - \theta_c\| \leq \|\theta_i - \theta_j\| \tag{6b}$$

for all $\theta_i, \theta_j \in S$. Furthermore, in (1a) and (1b), $w_i(t)$ and $w_o(t)$ are given by

$$w_i(t) = -\Delta A(q^{-1})z(t)$$

$$w_o(t) = \Delta B(q^{-1})z(t)$$

where for all times, $\|\Delta A\| < \delta$ and $\|\Delta B\| < \delta$. Therefore, we can also write

$$w(t) = \Delta\theta(t)^T\phi(t) \tag{7a}$$

where for all t,

$$\|\Delta\theta(t)\| < \delta. \tag{7b}$$

Next, we present the following result.

Lemma 1: Under assumptions 1)–5), there exists a constant $\epsilon_1 > 0$ such that for all k,

$$\Phi_k\Phi_k^T \geq \epsilon_1 I \tag{8a}$$

where

$$\Phi_k = [\phi(kN), \phi(kN+1), \cdots, \phi((k+1)N-1)]. \tag{8b}$$

Proof: See the Appendix.

Before we state the next result, the following notations are introduced. Define

$$\tilde{\theta}(k) = \hat{\theta}(k) - \theta^* \tag{9a}$$

where $\hat{\theta}(k)$ is the estimate of θ at time k, and

$$\theta^* = \begin{cases} \theta, & \text{defined by (2d) if the system is time invariant} \\ \theta_c, & \text{defined by (6b) if the system is time varying.} \end{cases} \tag{9b}$$

Lemma 2: For the adaptive scheme mentioned above, the parameter error θ_k satisfies the following recursion:

$$\tilde{\theta}((k+1)N) = (I + \sigma_0\Phi_k\Phi_k^T)^{-1}\tilde{\theta}(kN) + P(k+N-1)\sum_{i=kN}^{(k+1)N-1}\phi(i)w(i+1) \tag{10}$$

where $P(k)$ denotes the error covariance matrix at instant k, and $\sigma_0 I$ is the diagonal matrix to which $P(k)$ is periodically reset. Furthermore,

1) there exists a constant $\epsilon_2(\delta)$ such that

$$\left\| P(k+N-1) \sum_{i=1}^{(k+1)N-1} \phi(i)w(i+1) \right\| < \epsilon_2(\delta) \tag{11}$$

where

$$\epsilon_2(\delta) = \alpha_1 \delta, \qquad 0 < \alpha_1 < \infty$$

and

$$\lim_{\delta \to 0} \epsilon_2(\delta) = 0$$

2) $(I + \sigma_0 \Phi_k \Phi_k^T)^{-1}$ is a positive-definite matrix with all eigenvalues inside the unit circle.

Proof: By standard manipulations of the least squares algorithm with covariance resetting (LSCR), it is easy to show that

$$\tilde{\theta}(kN+1) = P((K+1)N-1)P(kN-1)^{-1}\tilde{\theta}(kN) + P((k+1)N-1)\sum_{i=kN}^{kN+1} \phi(i)w(i+1). \tag{12}$$

Next, (10) is obtained from (12) by using the relations

$$P(kN-1)^{-1} = (1/\sigma_0)I \tag{13a}$$

and

$$P((k+1)N-1)^{-1} = (1/\sigma_0)I + \Phi_k \Phi_k^T. \tag{13b}$$

Next, to prove (11) in the case of bounded disturbances, write the rightmost term in (10) as $P((k+1)N-1)\Phi_k W_k$ where

$$W_k = [w(kN), w(kN+1), \cdots, w((k+1)N-1)]^T.$$

The premultiplication and postmultiplication of (13b) by $P((k+1)N-1)$ yields

$$P((k+1)N-1)\Phi_k \Phi_k^T P((k+1)N-1) = P((k+1)N-1) - (1/\sigma_0)P((k+1)N-1)^2. \tag{14}$$

But from (13b) and Lemma 1, it is easy to show that

$$P((k+1)N-1)^2 \leq (\sigma_0^2/(\sigma_0\epsilon_1+1)^2)I.$$

Thus, (14) yields

$$P((k+1)N-1)\Phi_k \Phi_k^T P((k+1)N-1) \leq (\alpha_1^2)I$$

where

$$\alpha_1^2 = \frac{\sigma_0(1+\sigma_0\epsilon_1+\sigma_0)}{(1+\sigma_0\epsilon_1)^2}, \qquad 0 < \alpha_1^2 < \infty, \tag{15a}$$

i.e.,

$$\|P((K+1)N-1)\Phi_k W_k\| \leq \epsilon_2(\delta)$$

where

$$\epsilon_2(\delta) = \alpha_1 \delta. \tag{15b}$$

Next, for the case of time-varying parameters, in view of (7a), the rightmost quantity of (10) is bounded by

$$\left\| P((k+1)N-1)\Phi_k \Phi_k^T \right\| \, \|\Delta\theta(t)\|.$$

However, (13b) implies $P((k+1)N-1)\Phi_k \Phi_k^T < I$. Therefore, in view of (7b), the proof of (11) follows with $\epsilon_2(\delta)$ equal to δ.

Finally, to establish part 2), notice that Lemma 1 implies

$$(I + \sigma_0 \Phi_k \Phi_k^T)^{-1} < (1/(1+\sigma_0\epsilon_1))I.$$

This immediately establishes 2).

Lemma 3: For the LSCR algorithm described above, there exists an index k^* such that for all $k \geq k^*$,

1) $\|\tilde{\theta}(kN)\| \leq \epsilon_3(\delta)$

2) $\|\hat{\theta}((k+1)N) - \hat{\theta}(kN)\| \leq \epsilon_4(\delta)$ where $\epsilon_3(\delta)$ and $\epsilon_4(\delta)$ are nonnegative constants dependent on δ such that

$$\lim_{\delta \to 0} \epsilon_i(\delta) = 0, \qquad i = 3, 4.$$

Proof: To prove 1), we use Lemmas 1 and 2. Thus, (8b), (10), and (11) yield

$$\tilde{\theta}((k+1)N-1) \leq \alpha_2 \tilde{\theta}(kN) + \epsilon_2(\delta) \tag{16}$$

where

$$\alpha_2 = (1/(1+\sigma_0\epsilon_1)), \qquad 0 < \alpha_2 < 1. \tag{17}$$

$\epsilon_2(\delta)$ is given by (15b) and it goes to zero as $\delta \to 0$.

In view of (16) and (17), result 1) follows immediately. To establish 2), we make use of the result 1). Thus, (16) implies

$$\left\| \hat{\theta}((k+1)N) - \hat{\theta}(KN) \right\| \leq (1+\alpha_2) \left\| \tilde{\theta}(kN) \right\| + \epsilon_2(\delta)$$

which, in view of result 1), gives

$$\left\| \hat{\theta}((k+1)N) - \hat{\theta}(kN) \right\| \leq \epsilon_4(\delta)$$

where

$$\epsilon_4(\delta) = (1+\alpha_2)\epsilon_3(\delta) + \epsilon_2(\delta)$$

and it goes to zero as $\delta \to 0$. This concludes the proof of Lemma 3.

Finally, global stability in the presence of bounded disturbances and/or time-varying parameters is proved by the following.

Theorem 1: For the adaptive controller described above, in the presence of bounded disturbance and(or) slow time variation of parameters [subject to assumptions 1)–5)], there exists a sufficiently small positive constant δ^* such that for all $\delta \leq \delta^*$, the overall system remains globally stable.

Proof: Part 1) of Lemma 3 ensures that there exist finite positive constants $\epsilon_5(\delta)$ and $\epsilon_6(\delta)$ such that for all $k \geq k^*$,

$$\|A_k - A\| < \epsilon_5(\delta)$$
$$\|B_k - B\| < \epsilon_6(\delta)$$

where both $\epsilon_5(\delta)$ and $\epsilon_6(\delta)$ go to zero as $\delta \to 0$. Here, A_k and B_k denote estimates of A and B at the instant $t = kN$. Thus, assuming that δ is sufficiently small and the zeros of A^* lie within a disk of radius β, $0 < \beta < 1$, the pole placement equation guarantees that for all $k \geq k^*$, the zeros of $(L_k A + P_k B)$ lie within a disk of radius $\beta + \epsilon_7(\delta)$, $0 < (\beta + \epsilon_7(\delta)) < 1$. Here, $\epsilon_7(\delta)$ is such that $\lim_{\delta \to 0}\epsilon_7(\delta) = 0$.

In view of the above, the closed-loop system given by (5a) can be expressed in the following state-space form:

$$x(k+1) = F(k)x(k) + e(k) \tag{18}$$

where the eigenvalues of $F(k)$ lie within a disk of radius $\beta + \epsilon_7(\delta)$, $0 < (\beta + \epsilon_7(\delta)) < 1$, and $e(k)$ is uniformly bounded above for all k (because of the uniform boundedness of P_k, L_k, w_i, and w_o). Furthermore, in view of Lemma 3, there exists a constant $\epsilon_8(\delta)$ such that for all $k \geq k^*$,

$$\|F(k) - F(k-1)\| < \epsilon_8(\delta)$$

where $\epsilon_8(\delta)$ is such that $\lim_{\delta \to 0}\epsilon_8(\delta) = 0$.

Thus, provided δ is sufficiently small, we can use a result stated by Fuchs [19] to conclude that the unexcited system

$$x(k+1) = F(k)x(k)$$

is globally exponentially stable. This then implies that the system (18) is BIBS stable [9]. This concludes the proof of Theorem 1.

IV. Conclusion

This note presents the robustness analysis of a specific indirect adaptive pole placement algorithm. It is shown that provided random disturbances which affect the plant are small enough in magnitude, the global stability property of the basic scheme is retained. Furthermore, we also show that if the parameters of the plant vary sufficiently slowly with time within a compact set, the global stability property is not lost.

Appendix

In this Appendix, we present a proof of Lemma 1.

A. Presence of Bounded Random Noise

From (2e), (1a), and (1b) it is easily shown that

$$\phi(t) = M\psi(t) + d_1(t) \tag{A.1a}$$

where

$$\psi(t) = [z(t), z(t-1), \cdots, z(t-2n+1)]^T, \tag{A.1b}$$

$d_1(t)$ is some bounded signal vector, and $|d_1(t)|$ goes to zero as δ goes to zero. Also, M is the Sylvester resultant matrix of A and B, and it is nonsingular because of assumption 1). Thus, one can write

$$\Phi_k = M\Psi_k + D_{1k} \tag{A.2}$$

where Φ_k is given by (8b) and

$$\Psi_k = [\psi(kN), \psi(kN+1), \cdots, \psi((k+1)N-1)]. \tag{A.3}$$

Also, D_{1k} is some matrix with bounded entries and $\|D_{1k}\|$ goes to zero as δ goes to zero. Next, we proceed to show the validity of (8a).

Notice that over the interval $[kN, (k+1)N-1]$, the signal $z(t)$ satisfies (5a). Define

$$N_k(q^{-1}) = \sum_{i=0}^{d} n_i q^{-i} \tag{A.4}$$

$$= (L_k A + P_k B) \tag{A.5}$$

where $d \leq 2n$. Thus, (5a) can be rewritten as

$$N_k(q^{-1})z(t) = P_k r(t) + w_2(t) \tag{A.6}$$

where $w_2(t)$ is a bounded signal for all t, and it goes to zero as $w_i(t)$ and $w_o(t)$ go to zero. Next, define Q_k as the following $N \times (N-d)$ matrix:

$$Q_k = \begin{bmatrix} n_d & 0 & \cdots & 0 \\ \vdots & n_d & & \vdots \\ n_0 & \vdots & & n_d \\ 0 & n_0 & & \vdots \\ & 0 & & n_0 \\ \vdots & & & 0 \\ & \vdots & & \vdots \\ 0 & 0 & & 0 \end{bmatrix}. \tag{A.7}$$

Notice that the entries of Q_k are bounded because L_k and P_k have bounded coefficients. Furthermore, provided N_k is bounded away from zero, Q_k will be a full rank matrix because of its specific structure. If we designate λ_q as the maximum eigenvalue of $Q_k Q_k^T$, then we can write

$$\Phi_k \Phi_k^T \geq (1/\lambda_q)\Phi_k Q_k Q_k^T \Phi_k^T.$$

Thus, to prove (8a), it is sufficient to show that there exists some constant $\epsilon_3 > 0$ such that for all k,

$$[\Phi_k Q_k][\Phi_k Q_k]^T \geq \epsilon_3 I. \tag{A.8}$$

It is clear that for (A.8) to hold, one must choose $N - d \geq 2n$, i.e., $N \geq 4n$. Next, in view of (A.2), we can write

$$\Phi_k Q_k = M\Psi_k Q_k + D_{1k} Q_k \tag{A.9}$$

where, in light of the arguments given above, $D_{1k}Q_k$ is bounded and $\|D_{1k}Q_k\| \to 0$ as w_i and w_o go to zero. Next, consider $M\Psi_k Q_k$. Notice that M is a nonsingular matrix and $\Psi_k Q_k$ can be rewritten as

$$\Psi_k Q_k = \begin{bmatrix} r_1(kN+d) & r_1(kN+d+1) & \cdots & r_1(kN+N-1) \\ r_1(kN+d-1) & r_1(kN+d) & \cdots & r_1(kN+N-2) \\ \vdots & \vdots & & \vdots \\ r_1(kN+d-2n+1) & & & r_1(kN+N-2n) \end{bmatrix} + D_{2k} \tag{A.10}$$

where D_{2k} is a bounded matrix and $\|D_{2k}\|$ goes to zero as δ goes to zero. Also, $r_1(t)$ is given by

$$r_1(t) = P_k r(t), \qquad t \in [kN, (k+1)N-1]. \tag{A.11}$$

Notice that P_k can annihilate at most n complex exponential components in $r(t)$. Thus, for some integer, $m_1 \geq m - n$, we have

$$r(t) = \sum_{i=1}^{m_1} J_i \rho_i^t, \qquad t \in [kN, (k+1)N-1]$$

where $|\rho_i| = 1$. Thus, (A.10) can be rewritten as [15]

$$\Psi_k Q_k = E_k + D_{2k} \tag{A.12}$$

where E_k is factorizable in the following form:

$$E_k = G_{1k}\,\mathrm{diag}\,(J_i) G_{2k}. \tag{A.13}$$

The matrices G_{1k} and G_{2k} are given by

$$G_{1k} = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \rho_1^{-1} & \rho_2^{-1} & \cdots & \rho_{m_1}^{-1} \\ \vdots & \vdots & \vdots & \vdots \\ \rho_1^{-2n+1} & \rho_2^{-2n+1} & \cdots & \rho_{m_1}^{-2n+1} \end{bmatrix} \tag{A.14}$$

$$G_{2k} = \begin{bmatrix} \rho_1^{kN+d} & \rho_1^{kN+d+1} & \cdots & \rho_1^{kN+d+N-1} \\ \rho_2^{kN+d} & \rho_2^{kN+d+1} & \cdots & \rho_2^{kN+d+N-1} \\ \vdots & & & \vdots \\ \rho_{m_1}^{kN+d} & \rho_{m_1}^{kN+d+1} & \cdots & \rho_{m_1}^{kN+d+N-1} \end{bmatrix}. \tag{A.15}$$

Notice that we have already chosen $N \geq 4n$. Thus, because of the Vandermonde matrix structure of G_{1k} and G_{2k}, these are guaranteed to be of full rank provided $m_1 \geq 2n$. The choice of m, given in assumption 1), assures this. This then guarantees the existence of a constant $\epsilon_4 > 0$ such that for all k,

$$E_k E_k^T \geq \epsilon_4 I. \tag{A.16}$$

Finally, in view of (A.9) and (A.12), we have

$$[\Phi_k Q_k][\Phi_k Q_k]^T = [ME_k + D_k][ME_k + D_k]^T \tag{A.17}$$

where

$$D_k = MD_{2k} + D_{1k}Q_k \tag{A.18}$$

is a bounded matrix and $\|D_k\|$ goes to zero as δ goes to zero. Thus, choosing δ to be sufficiently small, (A.16) and the nonsingularity of M immediately prove (A.8).

B. Slow Time Variation of System Parameters

Following similar arguments as in Subsection A, we have once again

$$\phi(t) = M_1\psi(t) \tag{A.19}$$

where M_1 is the Sylvester resultant matrix of $(A + \Delta A)$ and $(B + \Delta B)$. Notice that in view of assumption 5), M_1 is always nonsingular. Thus, the rest of the proof follows exactly as in Subsection A.

References

[1] K. J. Astrom, "Analysis of Rohrs counter example to adaptive control," in *Proc. 22nd IEEE Decision Contr. Conf.*, Dec. 1983, pp. 982-987.

[2] P. A. Ioannou and J. Sun, "Theory and design of robust direct and indirect adaptive control schemes," *Int. J. Contr.*, vol. 42, pp. 775-813, 1988.

[3] K. S. Narendra and A. M. Annaswamy, "A new adaptive law for robust adaptation without persistent excitation," *IEEE Trans. Automat. Contr.*, vol. AC-32, pp. 134-145, 1987.

[4] P. A. Ioannou and P. V. Kokotovic, "Adaptive systems with reduced models," in *Lecture Notes in Control and Information Sciences, Vol. 47.* New York: Springer-Verlag, 1983.

[5] H. Elliott and G. C. Goodwin, "Adaptive implementation of the internal model principle," Univ. Massachusetts, Amherst, Tech. Rep. UMASS-ECE-MA84-1, Mar. 1984; also, *Proc. 23rd IEEE Decision Contr. Conf.*, 1984, pp. 1292-1297.

[6] R. Ortega and T. Yu, "Theoretical results on robustness of direct adaptive controllers: A survey," in *Proc. 10th IFAC World Congr.*, Munich, West Germany, 1987.

[7] L. Praly and R. Ortega, "Robust model reference adaptive controllers, Part 1: Stability analysis," in *Proc. 23rd IEEE Decision Contr. Conf.*, 1984.

[8] R. Cristi, "A globally stable adaptive control algorithm for discrete time systems with fast unmodeled dynamics," Univ. Michigan, Dearborn, Tech. Rep. UMD-UMASS-AFOSR-SEPT-83, Sept. 1983.

[9] B. D. O. Anderson and R. A. Johnstone, "Robust Lyapunov stability results and adaptive systems," in *Proc. 20th IEEE Decision Contr. Conf.*, Dec. 1981, pp. 510-515.

[10] ——, "Adaptive systems and time varying plants," *Int. J. Contr.*, vol. 37, no. 2, pp. 367-377, 1983.

[11] R. L. Kosut, C. R. Johnson, and B. D. O. Anderson, "Conditions for local stability and robustness of adaptive control systems," in *Proc. 22nd IEEE Decision Contr. Conf.*, Dec. 1983.

[12] R. L. Kosut and C. R. Johnson, "An input-output view of robustness in adaptive control," *Automatica*, Nov. 1983.

[13] H. Elliott, M. Das, and G. Ruiz, "A frequency domain analysis of direct adaptive pole placement algorithms in presence of unmodeled dynamics," in *Proc. 1983 IFAC Workshop Adaptive Contr. Signal Processing*, San Francisco, CA, June 1983.

[14] B. D. O. Anderson *et al.*, *Stability of Adaptive Systems: Passivity and Averaging Analysis.* Cambridge, MA: M.I.T. Press, 1986.

[15] H. Elliott, R. Cristi, and M. Das, "Global stability of adaptive pole placement algorithms," *IEEE Trans. Automat. Contr.*, Apr. 1985.

[16] G. C. Goodwin and E. K. Teoh, "Persistency of excitation in the presence of possibly unbounded signals," *IEEE Trans. Automat. Contr.*, June 1985.

[17] G. C. Goodwin, J. P. Norton, and M. N. Viswanathan, "Persistency of excitation of nonminimal models of systems having purely deterministic disturbances," *IEEE Trans. Automat. Contr.*, June 1985.

[18] G. C. Goodwin and K. S. Sin, *Adaptive Filtering Prediction and Control.* Englewood Cliffs, NJ: Prentice-Hall, 1984.

[19] J. J. Fuchs, "On the good use of the spectral radius of a matrix," *IEEE Trans. Automat. Contr.*, pp. 1134-1135, Oct. 1982.

[20] C. A. Desoer and M. Vidyasagar, *Feedback Systems: Input-Output Properties.* New York: Academic, 1975.

[21] M. Das and R. Cristi, "Robust adaptive pole placement in presence of bounded disturbances and slow time variation of parameters," in *Proc. ISA Conf.*, 1986, pp. 555-566.

On the Stability of Uncertain Polynomials with Dependent Coefficients

L. R. PUJARA

Abstract—**The thrust of the main theorem of the paper is to give a sufficient condition for reducing the conservatism of the stability bounds for a family of polynomials with dependent coefficients, including nonlinear coefficients. It is also proved that if a finite family of stable polynomials has the same even part, then the polynomial with the even part and the odd part formed by adding any positive multiple of the even parts and odd parts, respectively, of the given family is also stable. A similar result holds if the given family of polynomials has the same odd part.**

I. Introduction

Since the introduction of Kharitonov's theorem by Barmish [1] to the Western research literature, there has been a flurry of research activity in the area of stability of polynomials with uncertain coefficients and [3], [6]-[10] is just a partial list to indicate this activity. We know that Kharitonov's theorem gives conservative stability bounds for parameters of polynomials with dependent coefficients. Recently, Wei and Yedavalli [10] suggested a method for reducing this conservatism by multiplying the even and odd parts of a family of polynomials by suitable positive continuous functions of the parameters. Here, we have generalized this idea, viz. if the even and odd parts of an uncertain polynomial with dependent coefficients can be decomposed into subparts such that each subpart of the even part forms a positive pair with each subpart of the odd part, then the subparts of the even and odd parts as outlined might be multiplied by suitable positive continuous functions of the parameters to reduce the stability bounds for the parameters.

II. Definitions and Preliminaries

Definition 2.1: A polynomial of the form

$$f(s, q) = \alpha_0(q)s^n + \alpha_1(q)s^{n-1} + \cdots \alpha_{n-1}(q)s + \alpha_n(q) \tag{1}$$

with the vector q in Q, Q a compact subset of a Euclidean space of appropriate dimension, is called an uncertain polynomial. Q is called the uncertainty subset and $q \in Q$ is called the uncertainty vector. $\alpha_i(q)$, $0 \le i \le n$ are continuous functions of the uncertainty vector q.

The well-known classical Routh-Hurwitz test for the stability of polynomials becomes quite impractical if the number of polynomials is infinite, and this will be the case with uncertain polynomials with independent or dependent coefficients. Thus, Kharitonov's theorem becomes quite a valuable practical tool for dealing with such polynomials.

It is well known that in practical problems of control systems, one encounters characteristic polynomials with dependent coefficients. It is also well known that when Kharitonov's theorem is used for finding the stability bounds for parameters in the case of polynomials with dependent coefficients, one gets, in general, conservative bounds. Recently, Wei and Yedavalli [10] have proposed a method for reducing this conservatism for stability bounds, but it can be used only for simple examples. Thus, there is a need to establish new techniques to reduce this conservatism.

Definition 2.2: Two polynomials $h(p)$ and $g(p)$ with real coefficients and of degree m (or one of degree m and the other of degree $m-1$) form a positive pair if their roots $p_1, p_2, \cdots, p_m$ and $p'_1, p'_2, \cdots, p'_m$ (or $p'_1, p'_2, \cdots, p'_{m-1}$) are all distinct, real, and negative and they alternate as follows:

$$p'_1 < p_1 < p'_2 < p_2 < \cdots < p'_m < p_m < 0$$
$$(\text{or } p_1 < p'_1 < p_2 < p'_2 < \cdots < p'_{m-1} < p_m < 0)$$

and their leading coefficients are of the same sign.

Manuscript received October 14, 1988; revised November 2, 1988 and July 11, 1989.

The author is with the Department of Electrical Engineering, Wright State University, Dayton, OH 45435.

IEEE Log Number 9035074.

0018-9286/90/0600-0756$01.00